Specific heat in the integer quantum Hall effect: An exact diagonalization approach

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Abstract

We have studied the integer quantum Hall effect at finite temperatures by diagonalizing a single body tight binding model Hamiltonian including Aharonov-Bohm phase. We have studied the energy dependence of the specific heat and the Hall conductivity at a given temperature. The specific heat shows a sharp peak between two consecutive Hall plateaus. At very low temperatures, the numerical results of the temperature variations of specific heat (in the plateau region) are in good agreement with the analytical results.

PACS number(s): 73.40.Hm, 71.30.+h

Key Words: Integer quantum Hall effect; Specific heat; Tight binding model

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Quantum Hall effect continues to draw considerable interest since its surprised discovery [1] in two dimensional electronic systems at very low temperatures and in the presence of very high magnetic fields. In this phenomenon, the Hall conductivity gets quantized at integer filling factors ν with the value $\nu e^2/h$ in high accuracy and the diagonal conductivity vanishes at the same time. A similar phenomenon also occurs at certain fractional filling factors as well [2]. In this paper, however, we are concerned only with the integer quantum Hall effect (IQHE).

The scaling theory [3] predicts that all the states are localized in two dimensions due to disorder potential which respects time reversal symmetry. In the presence of strong magnetic field, perpendicular to the plane of the system, the electronic energy spectrum of the system shows a series of Landau bands. Since the magnetic field destroys time reversal symmetry and suppresses back scattering, some of the states appear to be extended at the center of each Landau band. On the other hand, the states away from the center of each band remain localized. The transport properties (electronic conduction) of the system do depend on the distribution of localized and extended states. This distribution of localized and extended states gives rise to IQHE. So long as the Fermi level lies in the region of localized states, the Hall conductivity remains pinned to a value forming a plateau. The critical transition [4] from one plateau to another occurs when Fermi level passes through the region of extended states.

On the other hand, the equilibrium properties (say specific heat) of the system depend simply on the density of states [5,6]. These properties cannot distinguish the localized states from extended states. This is indeed so, as it emerges from our systematic numerical diagonalization study in a lattice model. Agreeably, the separation of electronic specific heat from lattice specific heat in quantum Hall systems, from the experimental data, are extremely difficult. However, the experiments [5,7] are performed in multilayer systems to extract electronic specific heat.

In this paper, we consider a single body (spinless) tight binding model Hamiltonian [8] with random impurities in each site in the presence of a uniform magnetic field perpendicular

to the lattice plane. We have calculated the Hall conductivity σ_H and the specific heat C_v at a finite temperature T in a finite lattice. We show the formation of Hall plateau and the transition between two plateaus of the Hall conductivity by studying the energy variation of the Hall conductivity. When the Fermi level lies between two Landau bands, C_v is exponentially small since in such a case the probability of low energy excitation is very small. In such a region we find $C_v \sim (1/T^2)e^{-\Delta/T}$, where T is the temperature and Δ is a typical interband excitation gap. This behavior of C_v is consistent with the analytical results obtained from the self consistent Born approximation. On the other hand, when Fermi level lies within a Landau band C_v increases substantially and shows a sharp peak. This peak position of C_v lies between two consecutive Hall plateaus. We show analytically that C_v in this region is, in fact, proportional to the density of states at Fermi energy. C_v reflects the profile of the density of states. The width of the Hall conductivity plateau is larger than that of C_v plateau. This is due to the fact that while the transport properties distinguish between localized and extended states, the equilibrium properties do not.

Single body two dimensional tight binding Hamiltonian is given by

$$H = -\sum_{<ij>} \left(e^{ia_{ij}} C_i^\dagger C_j + h.c. \right) + \sum_i w_i C_i^\dagger C_i \tag{1}$$

where $C_i^\dagger$ and C_i are spinless Fermionic creation and destruction operators respectively at lattice site i. Here $< ij >$ refers to nearest neighbour hopping, and the hopping parameter is taken to be unity. A uniform magnetic field B is applied such that it produces a uniform magnetic flux $\phi = (hc/e)/M$ (M integer) per plaquette. When a particle hops from site j to site i, it acquires a phase $a_{ij} = \int_j^i \vec{A} \cdot \vec{dl}$, where the vector potential in the Landau gauge is taken to be $\vec{A} = (0, Bx)$. For this choice of gauge, an electron gets nonzero phase while hopping along y direction only. The (diagonal) second term in the Hamiltonian corresponds to random disorder potential at site i with strength w_i varying from $-W/2$ to $W/2$. We have constructed the Hamiltonian (1) by considering the periodic boundary condition. As a result, the energy spectrum becomes M number of Landau bands. Each band contains L^2/M states in a square lattice of size $L \times L$.

We compute the Hall conductivity σ_H using the Kubo formula [9]

$$\sigma_H = \frac{ie^2\hbar}{L^2}\sum_m \sum_{n\neq m} \frac{(f_n - f_m) < m|v_x|n >< n|v_y|m >}{(E_m - E_n)^2} \ , \tag{2}$$

where E_n is the energy of $|n>$-th state, and

$$v_\tau = (i/\hbar)\sum_j \left(C^\dagger_{j+\tau} C_j e^{ia_{j+\tau,j}} - C^\dagger_j C_{j+\tau} e^{-ia_{j+\tau,j}} \right) \tag{3}$$

is the velocity operator [10] along direction $\tau = (\hat{x} \text{ or } \hat{y})$. And $f_n = 1/(\exp[(E_n - E_F)/k_BT] + 1)$ is the Fermi function at temperature T with Fermi energy E_F. The electronic specific heat per unit area of the system can be computed using the formula [11]:

$$C_v = \frac{1}{L^2 k_B T^2} \sum_n (E_n - E_F)^2 f_n(1 - f_n) \ . \tag{4}$$

Figure 1 shows the energy variations of σ_H and C_v. These are computed using the formulae (2–4) for a system of size 12×12, strength of random potential $W = 0.5$, flux per plaquette $\phi = (hc/e)/12$, and at small temperature $k_BT = 0.01$. Three distinct IQHE plateaus, between four lowest Landau bands, at $\sigma_H = \nu(e^2/h)$ $(\nu = 1, 2, 3)$ are shown for $E_F < 0$. It is observed that the higher plateaus are formed at relatively lower values [12] compared to corresponding integral multiple of e^2/h. The decrease in the value of quantization is due to the relatively higher values of impurity strength for the formation of the corresponding plateaus [12]. The transitions between plateaus occur at the centers of the Landau bands. In these transition regions, C_v shows sharp but finite peaks with the width same as the Landau band width. In between the Landau bands, C_v is exponentially small since the low energy excitations are the least probable and the plateaus in C_v are also formed. The widths of plateaus in σ_H are relatively higher than that of the corresponding plateaus in C_v. This is because of the fact that the transport properties (like σ_H) are sensitive to extended states or localized states which are formed at the tail of the Landau bands but the equilibrium properties (like C_v) are insensitive to the nature of the states. This is indeed the case since the expression (4) for the specific heat does not depend on the characteristics (localized or extended) of the states.

Figure 2 shows the temperature variation of the specific heat at two different energies corresponding to two different plateaus (see Fig.1), keeping other parameters fixed. The best fitted form of C_v is found to be $C_v \sim (1/T^2)e^{-\Delta/T}$ which is consistent also with the numerical diagonalization result [13] for fractional quantum Hall states. Here Δ is a typical excitation gap which we find to be consistent with the Landau band gaps. The values of Δ for these two curves are found to be almost same while the prefactors are different.

We now analytically evaluate the specific heat at the regions of the plateaus and the transition between the plateaus separately at low temperatures by using self consistent Born approximation (SCBA) for the impurity potential, and compare those with the numerical results. The density of states of an IQHE system in SCBA [14] is given by

$$\rho(\epsilon) = \frac{1}{2\pi l^2}\frac{2}{\pi\Gamma^2}\sum_n [\Gamma^2 - (\epsilon - \epsilon_n)^2]^{1/2} \, , \tag{5}$$

which is a series of semicircular bands of radius Γ at Landau levels with energy $\epsilon_n = (n + \frac{1}{2})\hbar\omega_c$. Here ω_c is the cyclotron frequency and l is the magnetic length of the system. We assume that the broadenning 2Γ of each Landau level is very small compared to $\hbar\omega_c$, i.e., $\Gamma << \hbar\omega_c$. The electronic specific heat of the system can be expressed as

$$C_v = \frac{1}{k_B T^2}\int_0^\infty \rho(\epsilon)(\epsilon - E_F)^2 f(\epsilon)[1 - f(\epsilon)]d\epsilon \, , \tag{6}$$

where $f(\epsilon) = 1/(e^{[(\epsilon - E_F)/k_B T]} + 1)$ is the Fermi function.

(i) Specific heat at the plateau regions: The specific heat can be expressed using eqns (5) and (6) as

$$C_v = \frac{C}{k_B T^2}\sum_n \int_{\epsilon_n - \Gamma}^{\epsilon_n + \Gamma} (\epsilon - E_F)^2 \left[\Gamma^2 - (\epsilon - \epsilon_n)^2\right]^{1/2} f(\epsilon)[1 - f(\epsilon)]d\epsilon \, , \tag{7}$$

with the constant $C = 1/(\pi^2 l^2 \Gamma^2)$. We now consider that the Fermi energy E_F is in between two consecutive Landau bands. In this case an integer number of bands are filled. By Taylor's expansion of eqn (7) in powers of $\Gamma/\hbar\omega_c$, we obtain

$$C_v \approx \frac{C\pi\Gamma^2}{k_B T^2}\left[\sum_n (\epsilon_n - E_F)^2 f(\epsilon_n)[1 - f(\epsilon_n)] + \mathcal{O}\left(\frac{\Gamma}{\hbar\omega_c}\right)^2\right] \, . \tag{8}$$

At low temperatures ($\hbar\omega_c/k_BT >> 1$), we perturbatively evaluate C_v with the expansion parameter [15] $e^{-\hbar\omega_c/k_BT}$. We thus find

$$C_v(T) = \frac{4}{\pi l^2 k_B}\left[E_F^2 + \frac{\omega_c^2 N^2}{4} - E_F\omega_c N\right]\frac{1}{T^2}e^{-\hbar\omega_c/2k_BT} \ , \tag{9}$$

where N is the number of filled Landau bands. Note that the specific heat is activated through the gap between two Landau levels. Interestingly, this is the same temperature dependent form of C_v that we have found from our numerical diagonalization study. The prefactor in Eqn (9) depends on the value of Fermi energy and the number of filled Landau bands. For this reason, we obtain two different curves for two different Fermi energies in Fig. 2.

(ii) Specific heat at the regions of transition between two consecutive plateaus: In these regions, Fermi energy lies inside a Landau band. The approximations made in (i) will not hold good in this case since closeby states above the Fermi energy are available for low energy excitations. Using Sommerfeld's low temperature expansion [9] of the energy derivative of Fermi function in eqn (6), we obtain

$$C_v = \frac{\pi^2}{3}k_B^2 T\rho(E_F) \ . \tag{10}$$

Clearly, the specific heat in the transition regions follows the profile of the density of states which has maxima at the centers of the Landau bands. This is indeed so that we have also obtained this type of profile from our numerical study. However, the linear temperature dependence that we have obtained in eqn (10) does not resemble with our numerical calculations. In our numerical study, we have obtained a fitted form $C_v \sim (1/T^2)e^{-\Delta/T}$ here as well, with Δ much less than that in the case when Fermi energy lies between two successive Landau bands. We remark that this fitted form, in our numerical study, is a consequence of finite size diagonalization study since in such a case, there always remains small but finite gap between the states within a Landau band. In other words, the linear temperature dependence of C_v in eqn (10), obtained from continuum description of the density of states

(Eqn (5)) within a Landau band, cannot be achieved by a finite size diagonalization study. However, we believe that in $L \to \infty$ limit, the linear temperature dependence of C_v may hold good.

In summary, we have studied the finite temperature Hall conductivity and the specific heat of an IQHE system by finite size diagonalization of a tight binding model Hamiltonian including Aharonov-Bohm phase. The energy variation of Hall conductivity and the specific heat have been studied at low temperatures. The specific heat also shows plateau together with the Hall conductivity, and gets sharply peaked between two consecutive plateaus. We have also calculated specific heat analytically in self consistent Born approximation. The temperature variation of specific heat (in the plateau region) agrees well with the numerical results.

Acknowledgment: Financial assistance from Jawaharlal Nehru Centre for Advanced Scientific Research, Bangalore, is gratefully acknowledged.

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Figure Captions

Fig. 1: Energy variations of σ_H (circles) and C_v (triangles), for $k_BT = 0.01$, $W = 0.5$ and $M = 12$. σ_H is shown in the unit e^2/h while C_v is in arbitrary unit.

Fig. 2: Temperature variations of C_v for two different energies corresponding to two different plateaus. (i) Circles represent $E_F = -3.0$ and (ii) Triangles represent $E_F = -2.2$. Solid lines represent the best fitted functional form $C_v \sim (1/T^2)e^{-\Delta/T}$. Here $W = 0.5$ and $M = 12$. Here, C_v is shown in arbitrary unit.

Fig. 1

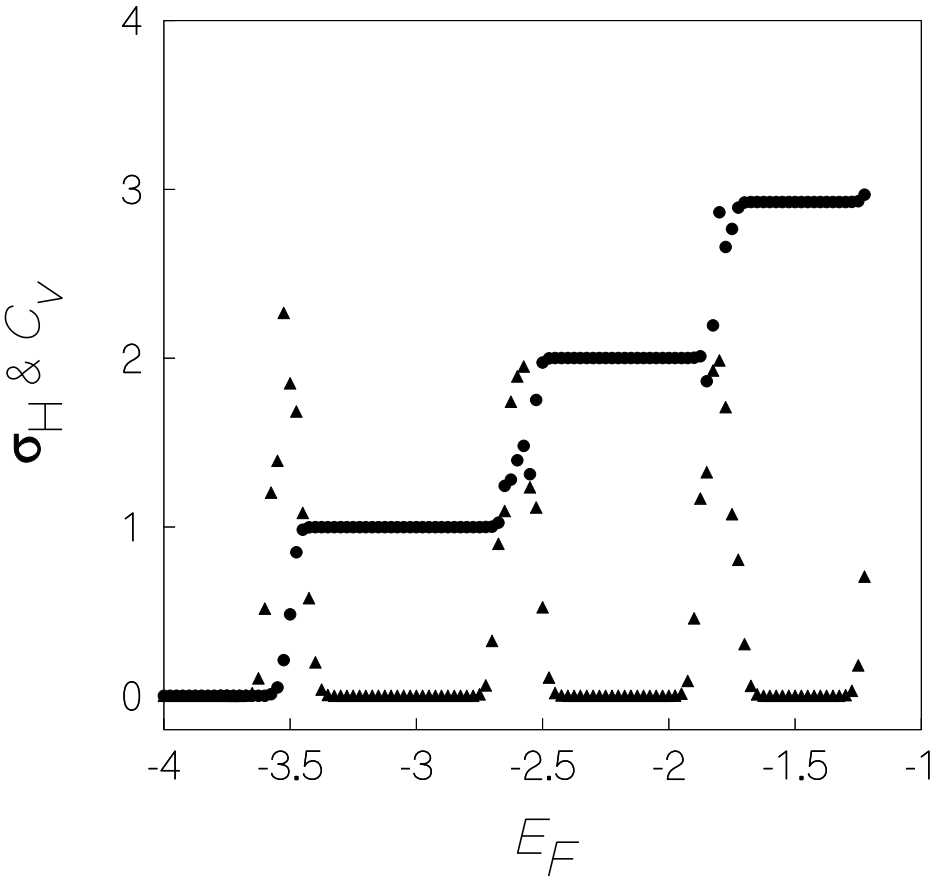

Fig. 2